Formula Sheet

Definition

Let C be a curve defined by a vector-valued function **r**, and assume that $\mathbf{r}'(t)$ exists when $t = t_0$. A tangent vector **v** at $t = t_0$ is any vector such that, when the tail of the vector is placed at point $\mathbf{r}(t_0)$ on the graph, vector **v** is tangent to curve *C*. Vector $\mathbf{r}'(t_0)$ is an example of a tangent vector at point $t = t_0$. Furthermore, assume that $\mathbf{r}'(t) \neq \mathbf{0}$. The **principal unit tangent vector** at *t* is defined to be

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}, \qquad (3.6)$$

provided $\|\mathbf{r}'(t)\| \neq 0$.

Theorem 3.4: Arc-Length Formulas

i. *Plane curve*: Given a smooth curve *C* defined by the function $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j}$, where *t* lies within the interval $[a, b]$, the arc length of *C* over the interval is

$$s = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|\ dt. \qquad (3.11)$$

Theorem 3.6: Alternative Formulas for Curvature

If *C* is a smooth curve given by $\mathbf{r}(t)$, then the curvature κ of *C* at *t* is given by

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}. \qquad (3.15)$$

If *C* is a three-dimensional curve, then the curvature can be given by the formula

$$\kappa = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}. \qquad (3.16)$$

Definition

Let *C* be a three-dimensional **smooth** curve represented by **r** over an open interval *I*. If $\mathbf{T}'(t) \neq \mathbf{0}$, then the principal unit normal vector at *t* is defined to be

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}. \qquad (3.18)$$

The binormal vector at *t* is defined as

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t), \qquad (3.19)$$

where $\mathbf{T}(t)$ is the unit tangent vector.

Definition

Let S be a surface defined by a differentiable function $z = f(x, y)$, and let $P_0 = (x_0, y_0)$ be a point in the domain of f. Then, the equation of the tangent plane to S at P_0 is given by

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0). \qquad (4.24)$$

Theorem 4.15: Directional Derivative of a Function of Three Variables

Let $f(x, y, z)$ be a differentiable function of three variables and let $\mathbf{u} = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$ be a unit vector. Then, the directional derivative of f in the direction of $\mathbf{u}$ is given by

$$\begin{aligned} D_{\mathbf{u}} f(x, y, z) &= \nabla f(x, y, z) \cdot \mathbf{u} \\ &= f_x(x, y, z)\cos\alpha + f_y(x, y, z)\cos\beta + f_z(x, y, z)\cos\gamma. \end{aligned} \tag{4.42}$$

Theorem 6.3: Evaluating a Scalar Line Integral

Let f be a continuous function with a domain that includes the smooth curve C with parameterization $\mathbf{r}(t),\ a \le t \le b$. Then

$$\int_C f\, ds = \int_a^b f(\mathbf{r}(t))\, \| \mathbf{r}'(t) \|\, dt. \tag{6.7}$$

Theorem 6.12: Green's Theorem, Circulation Form

Let D be an open, simply connected region with a boundary curve C that is a piecewise smooth, simple closed curve oriented counterclockwise (**Figure 6.33**). Let $\mathbf{F} = \langle P, Q \rangle$ be a vector field with component functions that have continuous partial derivatives on D. Then,

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C P dx + Q dy = \iint_D (Q_x - P_y) dA. \tag{6.13}$$

Definition

If $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field in $\mathbb{R}^3$ and $P_x, Q_y,$ and R_z all exist, then the **divergence** of **F** is defined by

$$\text{div}\, \mathbf{F} = P_x + Q_y + R_z = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}. \tag{6.16}$$

Definition

If $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field in $\mathbb{R}^3$, and $P_x, Q_y,$ and R_z all exist, then the **curl** of **F** is defined by

$$\begin{aligned} \text{curl}\, \mathbf{F} &= (R_y - Q_z)\mathbf{i} + (P_z - R_x)\mathbf{j} + (Q_x - P_y)\mathbf{k} \\ &= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}. \end{aligned} \tag{6.17}$$

Note that the curl of a vector field is a vector field, in contrast to divergence.

function	Laplace Transform
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$