Equations provided to students during the Material Science PhD qualifying exam:

Equation Number	Equation	Solving For
2.5c	$E = \int F dr$	Potential energy between two atoms
Section 2.5, Footnotes	$F = \frac{dE}{dr}$	Force between two atoms
2.6a	$E_A = -\frac{A}{r}$	Attractive energy between two atoms
2.6c	$E_R = \frac{B}{r^n}$	Repulsive energy between two atoms
Section 2.6	$F_A = \frac{1}{4\pi\varepsilon_0 r^2}(\lvert Z_1 \rvert e)(\lvert Z_2 \rvert e)$	Force of attraction between two isolated ions
2.8a	$\%\text{IC} = \{1 - \exp[-(0.25)(X_\text{A} - X_\text{B})^2]\} \times 100$	Percent ionic character

Equation Number	Equation	Solving For
3.4a	$a = 2R\sqrt{2}$	Unit cell edge length, FCC
3.4c	$\text{APF} = \frac{\text{volume of atoms in a unit cell}}{\text{total unit cell volume}} = \frac{Vs}{Vc}$	Atomic packing factor
3.4d	$a = \frac{4R}{\sqrt{3}}$	Unit cell edge length, BCC
3.5a	$\rho = \frac{nA}{V_C N_A}$	Theoretical density of a metal
3.8a	$q = \frac{P_x}{a}$	Point index referenced to *x* axis
3.9a	$u = n\left(\frac{x_2 - x_1}{a}\right)$	Direction index referenced to *x* axis
3.9d	$u = \frac{1}{3}(2U - V)$	Direction index conversion to hexagonal

3.9h	$U = n\left(\frac{a_1'' - a_1'}{a}\right)$	Hexagonal direction index referenced to a_1 axis (three-axis scheme)
3.10a	$h = \frac{na}{A}$	Planar (Miller) index referenced to *x* axis
3.11a	$\text{LD} = \frac{\text{number of atoms centered on direction vector}}{\text{length of direction vector}}$	Linear density
3.11c	$\text{PD} = \frac{\text{number of atoms centered on a plane}}{\text{area of plane}}$	Planar density

Equation Number	Equation	Solving For
4.2a	$N_v = N\ \exp\left(-\frac{Q_v}{kT}\right)$	Number of vacancies per unit volume
Section 4.2	$N = \frac{N_A \rho}{A}$	Number of atomic sites per unit volume
4.4a	$C_1 = \frac{m_1}{m_1+m_2} \times 100$	Composition in weight percent
4.4c	$C_1' = \frac{n_{m1}}{n_{m1}+n_{m2}} \times 100$	Composition in atom percent
4.4d	$C_1' = \frac{C_1 A_2}{C_1 A_2 + C_2 A_1} \times 100$	Conversion from weight percent to atom percent
4.4f	$C_1 = \frac{C_1' A_1}{C_1' A_1 + C_2' A_2} \times 100$	Conversion from atom percent to weight percent
4.4j	$C_1'' = \left(\frac{C_1}{\frac{C_1}{\rho_1}+\frac{C_2}{\rho_2}}\right) \times 10^3$	Conversion from weight percent to mass per unit volume
4.4l	$\rho_{\text{ave}} = \frac{100}{\frac{C_1}{\rho_1}+\frac{C_2}{\rho_2}}$	Average density of a two-component alloy
4.4n	$A_{\text{ave}} = \frac{100}{\frac{C_1}{A_1}+\frac{C_2}{A_2}}$	Average atomic weight of a two-component alloy

Equation Number	Equation	Solving For
5.3a	$J = \frac{M}{At}$	flux
5.3b	$J = -D\frac{dC}{dx}$	Fick's first law
5.4b	$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$	Fick's second law
5.4c	$\frac{C_x - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$	Solution to Fick's second law—for constant surface composition
5.5a	$D = D_0 \exp\left(-\frac{Q_d}{RT}\right)$	Temperature dependence of diffusion coefficient

6.12b	$\sigma_w = \frac{\sigma_y}{N}$	Safe (working) stress

Equation Number	Equation	Solving For
7.5a	$\tau_R = \sigma \cos \phi \cos \lambda$	Resolved shear stress
7.5c	$\tau_{\text{crss}} = \sigma_y(\cos \phi \cos \lambda)_{\max}$	Critical resolved shear stress
7.8a	$\sigma_y = \sigma_0 + k_y d^{-1/2}$	Yield strength (as a function of average grain size)—Hall-Petch equation
7.10a	$\%\text{CW} = \left(\frac{A_0 - A_d}{A_0}\right) \times 100$	Percent cold work
7.13a	$d^n - d_0^n = Kt$	Average grain size (during grain growth)

Equation Number	Equation	Solving For
8.5a	$\sigma_m = 2\sigma_0\left(\frac{a}{\rho_t}\right)^{1/2}$	Maximum stress at tip of elliptically shaped crack
8.5d	$K_c = Y\sigma_c\sqrt{\pi a}$	Fracture toughness
8.5e	$K_{Ic} = Y\sigma\sqrt{\pi a}$	Plane-strain fracture toughness
8.5f	$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}}$	Design (or critical) stress
8.5g	$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma Y}\right)^2$	Maximum allowable flaw size
8.7a	$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$	Mean stress (fatigue tests)
8.7b	$\sigma_r = \sigma_{\max} - \sigma_{\min}$	Range of stress (fatigue tests)
8.7c	$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$	Stress amplitude (fatigue tests)
8.7d	$R = \frac{\sigma_{\min}}{\sigma_{\max}}$	Stress ratio (fatigue tests)
Section 8.8	$\sigma = \frac{16FL}{\pi d_0^3}$	Maximum stress for fatigue rotating-bending tests
8.11a	$\sigma = \alpha_l E \Delta T$	Thermal stress
8.13a	$\dot{\epsilon}_s = K_1\sigma^n$	Steady-state creep rate (constant temperature)
8.13b	$\dot{\epsilon}_s = K_2\sigma^n \exp\left(-\frac{Q_c}{RT}\right)$	Steady-state creep rate
8.14a	$m = T(C + \log t_r)$	Larson-Miller parameter

Equation Number	Equation	Solving For
9.8b	$$W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L}$$	Mass fraction of liquid phase, binary isomorphous
9.8d	$$W_\alpha = \frac{C_0 - C_L}{C_\alpha - C_L}$$	Mass fraction of *α* solid-solution phase, binary isomorphous system
9.8e	$$V_\alpha = \frac{v_\alpha}{v_\alpha + v_\beta}$$	Volume fraction of *α* phase
9.8f	$$V_\alpha = \frac{\frac{W_\alpha}{\rho_\alpha}}{\frac{W_\alpha}{\rho_\alpha} + \frac{W_\beta}{\rho_\beta}}$$	For *α* phase, conversion of mass fraction to volume fraction
9.8h	$$W_\alpha = \frac{V_\alpha \rho_\alpha}{V_\alpha \rho_\alpha + V_\beta \rho_\beta}$$	For *α* phase, conversion of volume fraction to mass fraction
9.12b	$$W_e = \frac{P}{P+Q}$$	Mass fraction of eutectic microconstituent for binary eutectic system (per Figure 9.12.9)
9.12c	$$W_{\alpha'} = \frac{Q}{P+Q}$$	Mass fraction of primary *α* microconstituent for binary eutectic system (per Figure 9.12.9)
9.12d	$$W_\alpha = \frac{Q+R}{P+Q+R}$$	Mass fraction of total *α* phase for a binary eutectic system (per Figure 9.12.9)
9.12e	$$W_\beta = \frac{P}{P+Q+R}$$	Mass fraction of *β* phase for a binary eutectic system (per Figure 9.12.9)
9.19a	$$W_\rho = \frac{C_0' + 0.022}{0.74}$$	For a *hypo*eutectoid Fe-C alloy, the mass fraction of pearlite (per Figure 9.19.7)
9.19b	$$W_{\alpha'} = \frac{0.76 - C_0'}{0.74}$$	For a *hypo*eutectoid Fe-C alloy, the mass fraction of proeutectoid *α* ferrite phase (per Figure 9.19.7)
9.19c	$$W_\rho = \frac{6.70 - C_1'}{5.94}$$	For a *hyper*eutectoid Fe-C alloy, the mass fraction of pearlite (per Figure 9.19.7)
9.19d	$$W_{\mathrm{Fe_3C'}} = \frac{C_1' - 0.76}{5.94}$$	For a *hyper*eutectoid Fe-C alloy, the mass fraction of proeutectoid Fe_3C (per Figure 9.19.7)

Equation Number	Equation	Solving For
10.3c	$r^* = -\frac{2\gamma}{\Delta G_v}$	Critical radius for stable solid particle (homogeneous nucleation)
10.3d	$\Delta G^* = \frac{16\pi\gamma^3}{3(\Delta G_v)^2}$	Activation free energy for formation of stable solid particle (homogeneous nucleation)
10.3f	$r^* = \left(-\frac{2\gamma T_m}{\Delta H_f}\right)\left(\frac{1}{T_m - T}\right)$	Critical radius—in terms of latent heat of fusion and melting temperature
10.3g	$\Delta G^* = \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2}\right)\frac{1}{(T_m - T)^2}$	Activation free energy—in terms of latent heat of fusion and melting temperature
10.3k	$\gamma_{IL} = \gamma_{ST} + \gamma_{ST}\cos\theta$	Relationship among interfacial energies for heterogeneous nucleation
10.3l	$r^* = -\frac{2\gamma_{SL}}{\Delta G_v}$	Critical radius for stable solid particle (heterogeneous nucleation)
10.3m	$\Delta G^* = \left(\frac{16\pi\gamma_{SL}^3}{3\Delta G_v^2}\right) S(\theta)$	Activation free energy for formation of stable solid particle (heterogeneous nucleation)
10.3p	$y = 1 - \exp(-kt^n)$	Fraction of transformation (Avrami equation)
10.3q	$\text{rate} = \frac{1}{t_{0.5}}$	Transformation rate

Equation Number	Equation	Solving For
12.2a	$\rho = \frac{n'(\sum A_C + \sum A_A)}{V_C N_A}$	Density of a ceramic material
12.9a	$\sigma_{fs} = \frac{3F_f L}{2bd^2}$	Flexural strength for a bar specimen having a rectangular cross section
12.9b	$\sigma_{fs} = \frac{F_f L}{\pi R^3}$	Flexural strength for a bar specimen having a circular cross section
12.11a	$E = E_0(1 - 1.9P + 0.9P^2)$	Elastic modulus of a porous ceramic
12.11b	$\sigma_{fs} = \sigma_0\exp(-nP)$	Flexural strength of a porous ceramic

Equation Number	Equation	Solving For
14.5a	$\overline{M}_n = \sum x_i M_i$	Number-average molecular weight
14.5b	$\overline{M}_w = \sum w_i M_i$	Weight-average molecular weight
14.5c	$DP = \frac{\overline{M}_n}{m}$	Degree of polymerization
14.10a	$\overline{m} = \sum f_j m_j$	For a copolymer, average repeat unit molecular weight
14.11a	$\%\ \text{crystallinity} = \frac{\rho_c(\rho_s - \rho_a)}{\rho_s(\rho_c - \rho_a)} \times 100$	Percent crystallinity, by weight

Equation Number	Equation	Solving For
15.4a	$$E_r(t) = \frac{\sigma(t)}{\varepsilon_0}$$	Relaxation modulus
15.8a	$$TS = TS_\infty - \frac{A}{\overline{M}_n}$$	Polymer tensile strength